orders of magnitude higher. The optical signal was provided by a Bausch and Lomb 500-mm monochromator that was adjusted for a bandwidth of 60 Å. The photon flux density was monitored at the output of the monochromator to maintain a constant (i.e., independent from λ_0) light intensity. The photoconductive spectral measurements were performed at 85 and 300-K temperatures. The experimental results are shown in Figs. 3 and 4. We plotted the dc response (σ_{dc}) and microwave response ($\sigma_{\mu wave}$) in relative units with the peak of the response normalized to 1.

The results indicate that 1) the microwave response is sharper than its dc counterpart, and 2) the microwave peak response is shifted to shorter wavelengths compared to the dc peak by about 100–200 Å, depending on the crystal and the temperature. It may be noted that the change in the dc response with temperature is expected and reasonably well understood.

The measurements were reproducible over a long period of time (6 months) and were independent of the direction in which the optical wavelength was changed (i.e., from short to long wavelengths or vice versa). Corresponding dc and microwave measurements were always made at identical light intensities. The response spectra obtained for several different intensities of light and microwave fields (150–500 V/m) were found to be the same.

Discussion

The most probable cause for the observed shift of the photoconductive response peak is the inhomogeneity of the solid-state plasma.

The spectral distribution of photoconductivity for the dc case was first explained by DeVore [10], who solved the continuity equation for the spatial distribution of the electron density given below:

$$n(x, \lambda_0) = Ae^{-x/L_D} + Be^{x/L_D} + Ce^{-\alpha x} \tag{1}$$

where

L_D $=\sqrt{1/D\tau}=$characteristic diffusion length;
D diffusion constant;
τ carrier lifetime;
α $=\alpha(\lambda_0)=$optical absorption coefficient, a function of the wavelength λ_0.

A, B, and C are constants determined by the parameters D, τ, α, and S, the surface recombination velocity. In the dc measurement the applied electric field is uniform across the crystal and the photoconductive response is proportional to the average of the electron density:

$$\text{dc photoconductive response } \alpha \frac{1}{l}\int_0^l n(x, \lambda_0)\, dx$$

where l is the length of the crystal. The dc measurement is insensitive to the local variations of the electron density.

The microwave bridge produces standing waves and the field is nonuniform across the crystal. The response will be related to the product of the inhomogeneous electron density and the inhomogeneous electric field. Mathematically, one has to substitute (1) into the wave equation, solve for the propagation constant, and subsequently for the reflection efficient. Initial results of our calculation, which will be published shortly, show excellent agreement with the experiment.

Acknowledgment

The authors wish to thank Dr. R. Fischl, N. Jones, and J. Weiser for their help.

References

[1] T. S. Benedict and W. Shockley, *Phys. Rev.*, vol. 89, p. 1151, 1953.
[2] H. Jacobs *et al.*, "New microwave techniques in surface recombination and lifetime studies," *Proc. IEEE*, vol. 51, pp. 581–592, Apr. 1963.
[3] K. S. Champlin, D. B. Armstrong, and P. D. Gunderson, "Charge carrier inertia in semiconductors," *Proc. IEEE*, vol. 52, p. 677–685, June 1964.
[4] H. S. Sommers, Jr., "Demodulation of low-level broad-band optical signals with semiconductors," *Proc. IEEE*, vol. 51, pp. 140–146, Jan. 1963.
[5] H. S. Sommers, Jr., and E. K. Gatchell, "Demodulation of low-level broad-band optical signals with semiconductors," *Proc. IEEE*, vol. 54, pp. 1553–1568, Nov. 1966.
[6] W. D. Larson, K. M. VanVliet, R. N. Sharma, and L. H. Anderson, *Phys. Status Solidi*, vol. 35, p. k29, 1969.
[7] R. M. Arnold and F. J. Rosenbaum, "Microwave-biased photoconductive AM light detector," *IEEE Trans. Electron Devices*, vol. ED-18, pp. 81–87, Feb. 1971.
[8] J. Dziesiaty, *Phys. Status Solidi*, vol. 6, p. 913, 1964.
[9] K. S. Champlin, J. D. Holm, and D. B. Armstrong, "Analysis and calibration of a reflection coefficient bridge for use with any waveguide mode," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-15, pp. 477–478, Aug. 1967.
[10] H. B. DeVore, *Phys. Rev.*, vol. 102, p. 86, 1956.

Wave Propagation in Multilayered Drifted Solid-State Plasmas

A. C. BAYNHAM, A. D. BOARDMAN, M. R. B. DUNSMORE, AND J. TIERNEY

***Abstract*—The problem of the propagation of transverse magnetoplasma waves in drifted, stratified, media consisting of periodically distributed homogeneous slabs of solid-state plasmas is investigated. The action of the drifting carriers is to introduce an asymmetry into the propagation characteristics while the influence of the periodic structure introduces space harmonics. A detailed assessment of the possibility of a synchronous space-harmonic interaction for on-axis waves, which leads to wave amplification, has been performed. It is found that, contrary to the results of some prior work in the field, the gain that occurred could not be attributed directly to the periodic structure.**

I. Introduction

Over the last few years some considerable interest has developed in the propagation of electromagnetic waves in stratified media [1]–[4]. If the stratified medium is periodic and an electric field is applied to the system, there arises an attractive possibility of utilizing space harmonics to promote wave amplification (solid-state analog of the traveling-wave tube). The space harmonics provide the possibility of a synchronous interaction, in which gain occurs at low plasma velocities. A theoretical estimate, by Wisseman, of a single slab of plasma, supporting on-axis helicons under the influence of an electric and magnetic field [5], claims to reveal the possibility of helicon amplification due to the Fabry–Perot geometry. The purpose of this short paper is to determine, precisely, whether the presence of space harmonics arising from a drifted (presence of an external electric field) periodic structure will promote on-axis helicon amplification or not.

In this short paper, then, the type of electromagnetic wave transmission made possible by immersing the system in a uniform external magnetic field $\boldsymbol{B}_0$ that is parallel or has a component parallel to the direction of propagation [6], [7] is considered. The modes of propagation turn out to be circularly polarized magnetoplasma waves of which the now classical helicon is a special case. One of the special advantages of such a configuration is that the band edges are sensitive to the magnetic field.

II. Properties of the Layers

The propagation of an electromagnetic wave through an infinite periodic structure, along a direction that is parallel to a constant external magnetic field $(0, 0, B_0)$, is examined. This structure is made up of alternate homogeneous slabs of solid-state plasma of uniform thickness. The whole system is generated by repetition, along the direction of wave propagation, of a unit cell consisting of two such slabs of different thickness and containing different carrier densities. The whole system is immersed in a constant external electric field $\boldsymbol{E}=(0, 0, E)$.

If a layer contains α types of free carriers possessing a scalar mass m_α and a charge q_α, then the external electric field will impart drift velocities $\boldsymbol{V}_\alpha=(0, 0, V_\alpha)$ to them. The linearized equation of motion of the carriers then is

$$m_\alpha \left\{ \frac{\partial \boldsymbol{v}_\alpha}{\partial t} + V_\alpha \cdot \frac{\partial \boldsymbol{v}_\alpha}{\partial z} + \nu_\alpha \boldsymbol{v}_\alpha \right\} = q_\alpha(\boldsymbol{e} + \boldsymbol{V}_\alpha \times \boldsymbol{b} + \boldsymbol{v}_\alpha \times \boldsymbol{B}_0) \tag{1}$$

where $\boldsymbol{v}_\alpha$ is the ac particle velocity, $\boldsymbol{V}_\alpha$ is the dc drift velocity, ν_α is a constant phenomenological collision frequency, and $\boldsymbol{e}$ and $\boldsymbol{b}$ are, respectively, the electric and magnetic fields associated with the passage of the electromagnetic wave.

Assuming the field variables in each inhomogeneous layer to possess a temporal variation of $e^{-i\omega t}$, Maxwell's equations become

$$\text{curl}\, \boldsymbol{e} = i\omega \boldsymbol{b} \tag{2}$$

$$\text{curl}\, \boldsymbol{b} = \mu_0 \sum_\alpha n_\alpha q_\alpha \boldsymbol{v}_\alpha - i\frac{\omega}{c^2}\epsilon_L \boldsymbol{e} \tag{3}$$

Manuscript received January 10, 1972; revised August 3, 1972. This short paper was contributed by permission of the Director of the Royal Radar Establishment, and published by permission of the Controller of Her Brittanic Majesty's Stationery Office.

A. C. Baynham and M. R. B. Dunsmore are with the Royal Radar Establishment, Malvern, England.
A. D. Boardman and J. Tierney are with the Department of Pure and Applied Physics, University of Salford, Salford, England.

where μ_0 is the permeability of free space, n_α is the free carrier density, c is the velocity of light *in vacuo*, and ϵ_L is the lattice dielectric constant. If the waves in an infinite homogeneous medium are now assumed to propagate along the z axis with a z dependence of the form e^{ikz}, then (1), after separation into modes possessing opposite senses of circular polarization, has solutions as follows:

$$v_\alpha^\pm = \frac{q_\alpha}{m_\alpha} \frac{(\omega - kV_\alpha)e^\pm}{\omega(\nu_\alpha + ikV_\alpha - i\{\omega \pm \omega_{c\alpha}\})} = v_{x\alpha} \pm iv_{y\alpha} \tag{4}$$

where $\omega_{c\alpha} = q_\alpha B_0/m_\alpha$ is the cyclotron frequency of the α carriers and q_α is obviously $|q|$ or $-|q|$, where $|q|$ is the electronic charge. In addition, (3) gives

$$k^2 e^\pm = k^2(e_x \pm ie_y) = i\omega\mu_0 \sum_\alpha n_\alpha q_\alpha v_\alpha^\pm + \frac{\omega^2}{c^2}\epsilon_L e^\pm \tag{5}$$

so that, finally,

$$(k^2 - \omega^2/c^2\epsilon_L) = -\frac{1}{c^2}\sum_\alpha \frac{\omega_{p\alpha}^2(\omega - kV_\alpha)}{(-kV_\alpha + i\nu_\alpha + (\omega \pm \omega_{c\alpha}))} \tag{6}$$

where

$$\omega_{p\alpha}^2 = \frac{n_\alpha q_\alpha^2}{\epsilon_0 m_\alpha}$$

is called the plasma frequency. Equation (6) can now be used to analyze the periodic structure.

III. Dispersion Equation of a Drifted Periodic Structure

Since, for normal incidence, the modes of opposite polarization [6], [7] can be distinguished from each other, their behavior can be calculated separately in the periodic structure. Suppose the unit cell is composed of two layers whose thicknesses are l_1 and l_2, respectively, occupying the region $-l_1 < z < l_2$. The electric fields in the layers then are

$$-l_1 < z < 0: e(z) = (\sum_j A_j e^{i\alpha_j(\omega)z})e^{-i\omega t} \tag{7}$$

$$0 < z < l_2: e(z) = (\sum_j B_j e^{i\beta_j(\omega)z})e^{-i\omega t} \tag{8}$$

where the sum over j is over the roots of (6), i.e., $\alpha_j(\omega)$ and $\beta_j(\omega)$ are the roots of (6) for each layer. The latter equation, using the fact that, for carriers such as electrons or holes, $\omega_{c\alpha} \gg kV_\alpha$, is simply

$$k^2 - \frac{k}{c^2}\sum_\alpha \frac{V_\alpha \omega_{p\alpha}^2}{(\omega \mp \omega_{c\alpha} + i\nu_\alpha)} + \frac{\omega}{c^2}\sum_\alpha \frac{\omega_{p\alpha}^2}{(\omega \mp \omega_{c\alpha} + i\nu_\alpha)} = 0. \tag{9}$$

We now come to a feature that has not previously received much attention in this context, namely that the wavenumber k associated with the plane waves in each layer is of thef orm

$$k = -\delta \pm \gamma \tag{10}$$

so that the modulus of the forward wavenumber is not simply the same as the backward wavenumber. The drifting of the carriers actually removes the inversion symmetry. In fact, it is easy to show that this amounts to a Doppler shifting of the plasma wave frequency [8], [9], and is a well-established result for a homogeneous medium. We will now examine its impact on the usual derivation of the dispersion equation of a periodic structure.

The application of the boundary conditions to the drifted field solutions, i.e., continuity at the cell interface and the imposition of Floquet's theorem, i.e.,

$$e(l_2) = \exp\{\kappa(l_1 + l_2)\}e(-l_1) \tag{11}$$

leads to

$$A_1 + A_2 = B_1 + B_2 \tag{12}$$

$$\alpha_1 A_1 + \alpha_2 A_2 = \beta_1 B_1 + \beta_2 B_2 \tag{13}$$

$$A_1 e^{-i\alpha_1 l_1} + A_2 e^{-i\alpha_2 l_1} = (B_1 e^{i\beta_1 l_2} + B_2 e^{i\beta_2 l_2})e^{-i\kappa(l_1+l_2)} \tag{14}$$

$$\alpha_1 A_1 e^{-i\alpha_1 l_1} + \alpha_2 A_2 e^{-i\alpha_2 l_1} = (B_1\beta_1 e^{i\beta_1 l_2} + B_2\beta_2 e^{i\beta_2 l_2})e^{-i\kappa(l_1+l_2)}. \tag{15}$$

The determinant of the matrix operating on the vector $[{A_1 \atop B_1}]$ then gives the required relationship between ω and κ. If y is defined as $e^{i\kappa(l_1+l_2)}$, then after some laborious algebra it can be shown that, provided

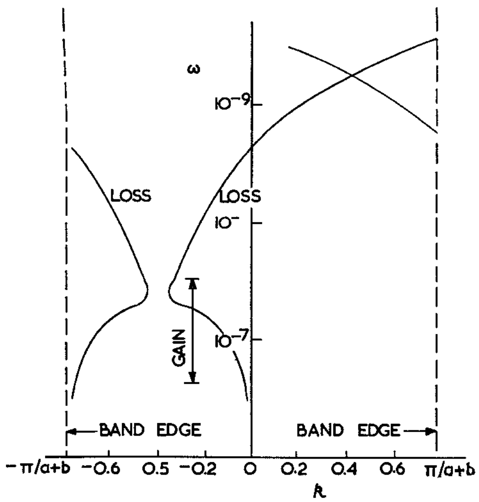

Fig. 1. Dispersion equation for a drifted InSb–Ge sandwich structure at 77 K (i.e., a plot of real frequency ω versus the real part of the wavenumber k). The areas marked "gain" or "loss" show regimes in which the imaginary part of the wavenumber is, respectively, greater than or less than **zero.**

$$\alpha_1 = -\delta_1 + \gamma_1$$
$$\alpha_2 = -\delta_1 - \gamma_1$$
$$\beta_1 = -\delta_2 + \gamma_2$$
$$\beta_2 = -\delta_2 - \gamma_2 \tag{16}$$

y is given by the solutions of

$$y^2 + by + c = 0 \tag{17}$$

where

$$b = -\frac{1}{2\gamma_1\gamma_2}\{((\gamma_1 + \gamma_2)^2 - (\delta_1 - \delta_2)^2)\cos(\gamma_2{}_2 + \gamma_1 l_1) + ((\delta_1 - \delta_2)^2 - (\gamma_1 - \gamma_2)^2)\cos(\gamma_1 l_1 - \gamma_2 l_2)\}e^{i(\delta_1 l_1 + \delta_2 l_2)} \tag{18}$$

$$c = e^{-2i(\delta_1 l_1 + \delta_2 l_2)}. \tag{19}$$

Because of the asymmetry it is not possible to identify $\cos\kappa(l_1 + l_2)$ with the sum of the roots of (19), as is usual in such a problem. It is necessary therefore to work in terms of y. In general, there is also an added complication because the existence of scattering makes y, b, and c complex. Therefore, we write these quantities as the complex numbers

$$y = P + iQ \tag{20}$$

$$b = b_r + ib_i \tag{21}$$

$$c = c_r + ic_i. \tag{22}$$

The real and imaginary parts of (17) then lead to

$$P = -\frac{(b_r Q + c_i)}{2Q + b_i} \tag{23}$$

and

$$\sum_{i=0}^{4} a_i Q^i = 0 \tag{24}$$

where

$$a_0 = b_i^2 c_r + c_i(c_i - b_i b_r) \tag{25}$$

$$a_1 = b_i(4c_r - b_i^2 - b_r^2) \tag{26}$$

$$a_2 = 4c_r - 5b_i^2 - b_r^2 \tag{27}$$

$$a_3 = -8b_i \tag{28}$$

$$a_4 = -4. \tag{29}$$

The real and imaginary parts of the wavenumber χ are

$$\kappa_r = \frac{1}{(l_1 + l_2)}\tan^{-1}\left(\frac{Q}{P}\right) \tag{30}$$

$$\kappa_i = -\frac{1}{2(l_1 + l_2)} \ln (P^2 + Q^2). \tag{31}$$

This problem has been solved numerically (e.g., Fig. 1), primarily in an attempt to see whether, for an electron-hole plasma system, the structure provides an additional space-harmonic coupling mechanism giving rise to amplification, i.e., a change in the sign of κ_i. We conclude that no gain can be directly attributed to the structure, although gain is evident when at least one of the constituent basis functions (infinite medium solutions) exhibits amplification. Coupling to space harmonics evidently requires off-axis propagation, since only in this manner can a longitudinal ac electric field be provided.

References

[1] T. Tamir, H. C. Wang, and A. A. Oliner, "Wave propagation in sinusoidally stratified deielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 323–335, May 1964.
[2] C. Yeh, K. F. Casey, and A. A. Kaprielian, "Transverse magnetic wave propagation in sinusoidally stratified dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 297–302, May 1965.
[3] J. T. Taub and J. Cohen, "Quasi-optical waveguide filters for millimeter and submillimeter wavelengths," *Proc. IEEE (Special Issue on Millimeter Waves and Beyond)*, vol. 54, pp. 647–656, Apr. 1966.
[4] K. F. Casey, J. R. Mathes, and C. Yeh, "Wave propagation in sinusoidally stratified plasma media," *J. Math. Phys.*, vol. 10, p. 891–897, May 1969.
[5] W. R. Wisseman, "Concerning helicon amplification in the Fabry-Perot geometry," *Phys. Lett.*, vol. 28A, p. 57–58, Oct. 1968.
[6] A. C. Baynham and A. D. Boardman, "Band theory of helicons," *Proc. Phys. Soc. London (Gen.)*, vol. 1, p. 363–366, May 1968.
[7] ——, "Transverse magneto-plasma propagation in a periodic structure," *Proc. Phys. Soc. London (Solid State Phys.)*, vol. 2, p. 619–628, Apr. 1969.
[8] D. J. Bartelink, "Propagation and instability of transverse waves in current-carrying electron-hole plasmas," *Phys. Rev.*, vol. 158, p. 400–414, June 1967.
[9] C. A. Nanney, "Helicon-drift current interaction in a solid state plasma waveguide," *Phys. Rev.*, vol. 138, p. 1484–1489, May 1965.

Variational Expressions, Pseudoenergy, and Power

P. DALY

***Abstract*—A variational expression involving the longitudinal fields in an inhomogeneous lossless waveguide, which acts as the starting point for numerical procedures, is shown to be equivalent to a simple relationship between power flow, stored energy, and phase velocity.**

Recently Chorney [1] and Chorney and Penfield [2] have developed several interesting results in various uniform waveguide systems concerning power flow and pseudoenergy. A particular result which has been derived for a lossless passive uniform waveguide relates the power flow P to the pseudoenergies U_T and U_z:

$$P - \frac{\omega}{\beta}(U_T - U_z) = 0 \tag{1}$$

where U_T and U_z are the energies stored per unit length in the transverse and longitudinal fields, respectively, and ω/β is the phase velocity. What is even more interesting about the above expression is that, at least for nondispersive isotropic materials it is a variational expression in the unknown waveguide fields and represents the starting point for several numerical solutions of the waveguide problem including the finite-element method.

Consider an inhomogeneous lossless uniform waveguide with field dependence $\exp[j(\omega t - \beta z)]$ whose field components may be defined [3], [4] in terms of two scalars ϕ and ψ as

$$H_z = \phi \tag{2}$$

$$E_z = \frac{\beta\psi}{\omega\epsilon_0} \tag{3}$$

$$k^2 H_t = -j\beta[\nabla_t\phi + \kappa(e_z \times \nabla_t\psi)] \tag{4}$$

$$k^2 E_t = j\omega\mu_0[\mu(e_z \times \nabla_t\phi) - \bar{\beta}^2\nabla_t\psi] \tag{5}$$

where κ and μ are relative dielectric constants and permeabilities, respectively.

$$k^2 = \left(\frac{\omega}{c}\right)^2 \kappa\mu - \beta^2 \qquad \bar{\beta} = \frac{\beta c}{\omega}. \tag{6}$$

Manuscript received February 11, 1972; revised July 10, 1972.
The author is with the Department of Electrical and Electronic Engineering University of Leeds, Leeds, England.

The total power flow over the waveguide cross section is

$$P = \frac{1}{2}\iint e_z \cdot (E_t \times H_t^*)\, dS$$
$$= \frac{\omega\mu_0\beta}{2}\iint \frac{1}{k^4}[\mu|\nabla_t\phi|^2 + \kappa\bar{\beta}^2|\nabla_t\psi|^2 + (\kappa\mu + \bar{\beta}^2)e_z \cdot (\nabla_t\psi \times \nabla_t\phi)]\, dS. \tag{7}$$

Defining the terms involving stored energy per unit length, we have longitudinal pseudoenergy:

$$U_z = \frac{1}{4}\iint (H_z^* B_z + E_z D_z^*)\, dS$$
$$= \frac{\mu_0}{4}\iint [\mu\phi^2 + \kappa\bar{\beta}^2\psi^2]\, dS \tag{8}$$

transverse magnetic pseudoenergy:

$$U_{mT} = \frac{1}{4}\iint H_t^* \cdot B_t\, dS$$
$$= \frac{\mu_0\beta^2}{4}\iint_S \frac{\mu}{k^4}[|\nabla_t\phi|^2 + 2\kappa e_z \cdot (\nabla_t\psi \times \nabla_t\phi) + \kappa^2|\nabla_t\psi|^2]\, dS \tag{9}$$

transverse electric pseudoenergy:

$$U_{eT} = \frac{1}{4}\iint_S E_t \cdot D_t^*\, dS$$
$$= \frac{\mu_0}{4}\left(\frac{\omega}{c}\right)^2 \iint_S \frac{\kappa}{k^4}[\mu^2|\nabla_t\phi|^2 - 2\mu\bar{\beta}^2 e_z \cdot (\nabla_t\phi \times \nabla_t\psi) + \bar{\beta}^4|\nabla_t\psi|^2]\, dS \tag{10}$$

transverse pseudoenergy:

$$U_T = U_{mT} + U_{eT}$$
$$= \frac{\mu_0}{4}\left(\frac{\omega}{c}\right)^2 \iint_S \frac{1}{k^4}[\mu(\kappa\mu + \bar{\beta}^2)|\nabla_t\phi|^2 + \kappa\bar{\beta}^2(\kappa\mu + \bar{\beta}^2)|\nabla_t\psi|^2 + 4\mu\kappa\bar{\beta}^2 e_z \cdot (\nabla_t\psi \times \nabla_t\phi)]\, dS. \tag{11}$$

Substituting for P, U_z, and U_T in (1) and rearranging, we obtain

$$\left(\frac{\omega}{c}\right)^2 \iint_S [\mu\phi^2 + \kappa\bar{\beta}^2\psi^2]\, dS - \iint_S \frac{\mu|\nabla_t\phi|^2 + \kappa\bar{\beta}^2|\nabla_t\psi|^2 + 2\bar{\beta}^2 e_z \cdot (\nabla_t\psi \times \nabla_t\phi)}{\kappa\mu - \bar{\beta}^2}\, dS = 0. \tag{12}$$

Expression (12) is, however, a variational expression in the scalar functions ϕ and ψ which was derived [4], [5] in connection with a finite-element analysis of hybrid-modes in waveguide. The result in [4] and [5] was derived for $\mu = 1$ but the generalization to (12) is simple.

As a further point of interest, it is worth noting that variational expressions are often written directly in terms of vector fields. Such an expression is to be found in the work of English [6], who derives the following vector variational expression [7]:

$$\beta\iint_S [H^* \cdot (e_z \times E) - E^* \cdot (e_z \times H)]\, dS - \omega\iint_S (E^* \cdot \varepsilon \cdot E + H^* \cdot \mu \cdot H)\, dS - j\iint_S [E^* \cdot (\nabla_t \times H) - H^* \cdot (\nabla_t \times E)]\, dS = 0 \tag{13}$$

subject to the boundary condition $n \times E = 0$.

Under the condition that the medium is isotropic and nondispersive, (13) can easily be shown by direct substitution from (2)–(5) to be another reformulation of (1). It is interesting to speculate that perhaps (1) remains a variational expression under conditions more stringent than those imposed here, e.g., when the medium is dispersive or anisotropic.

Finally, it is shown by Chorney that for propagating modes, total electric and magnetic stored energies are equal. Using the subscripts m and e to denote magnetic and electric, respectively, we ob-